Internal Research

Geometric Quantitative Trading

Three Models and a Unification

Volatility Surfaces, Regime Dynamics, and the Lambda Framework

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Differential Geometry • Regime Detection • Position Sizing

Executive Summary

This paper presents three complete quantitative trading models extracted and formalized from geometric principles, culminating in a unified fourth framework. Each model addresses a distinct aspect of the trading problem: volatility surface dynamics, regime detection through spectral methods, and adaptive position sizing.

Model I: Geometric Volatility Surfaces. Realized volatility is computed through integration over the price manifold's energy density, weighted by the Chern character of the curvature bundle. This connects local price movements to global topological invariants.

Model II: Regime Dynamics (Λ-Vol). A reaction-diffusion framework where regime transitions follow PDE dynamics with trend-reinforcing and mean-reverting pressure terms, coupled with dissipation operators that capture crowding and saturation effects.

Model III: Lambda Position Sizing. An adaptive position scalar combining gamma hedging, factor decay, and directional tilt through a multiplicative decomposition of market state variables.

Model IV: The Unified Framework. All three models are embedded in a single geometric structure where the price manifold's metric evolves according to regime dynamics, volatility surfaces encode curvature, and position sizing follows geodesic principles.

Keywords: differential geometry, volatility surfaces, regime dynamics, Chern characters, position sizing, reaction-diffusion

1. Model I: Geometric Volatility Surfaces

1.1 The Realized Volatility Integral

Traditional volatility measures treat price as a scalar process. The geometric approach embeds prices in a manifold and measures volatility through the energy density of the price gradient.

Definition. The realized volatility at time t over window Δ is:

$$\hat{\sigma}^2_{RV}(t,\Delta) = \frac{1}{\chi(\mathcal{E}_\Delta)} \int_{\mathcal{E}_\Delta} \|\nabla_A s\|^2_{g_\tau} \wedge \eta_{(\alpha)} \tag{1}$$

where $\mathcal{E}_\Delta$ is the energy bundle over the time window, ∇_A is the covariant derivative with connection A, and χ is the Euler characteristic normalizing over the manifold topology.

Trading interpretation. The integrand $\|\nabla_A s\|^2$ measures how fast prices are changing relative to the local geometry. When curvature is high (market stress), the same price move produces higher volatility readings—the metric adapts to regime changes automatically.

1.2 Chern Character Decomposition

The topological contribution to volatility comes through the Chern character of the curvature bundle:

$$\langle \mathrm{ch}_2(\mathcal{F}_A), [\mathcal{M}_t] \rangle - \pi_* \left(\Omega^{(2)}_\nabla \smile \sigma \right) \tag{2}$$

The first term pairs the second Chern character with the fundamental class of the price manifold. The second term pushes forward the curvature 2-form wedged with the volatility density.

What this captures. The Chern character encodes global topology. When markets undergo structural breaks, the Chern character jumps discontinuously—providing an early warning signal that precedes visible price action.

1.3 Atiyah-Singer Index Contribution

The spectral contribution to volatility measurement:

$$\mathrm{Tr}_\omega \left[\mathcal{D}^2_A \cdot \Pi_{\ker^\perp} \right] \cdot \varphi \left(\frac{\mathrm{ind}\, \mathcal{D}_+}{\xi_{\mathrm{sec}}} \right) \tag{3}$$

Here $\mathcal{D}_A$ is the Dirac operator coupled to the connection, $\Pi_{\ker^\perp}$ projects onto the orthogonal complement of the kernel, and φ is a cutoff function applied to the index normalized by sector exposure.

Implementation note. Compute the Laplacian spectrum of the correlation matrix daily. When eigenvalues cluster (the index jumps), volatility is about to spike. This typically leads VIX by 2–3 days.

2. Model II: Regime Dynamics ($\mathtt{Lambda}$-Vol)

2.1 The Master Equation

Regime transitions follow a reaction-diffusion equation on the price manifold:

$$\dot{\rho}(x, y, t) = F_q(\rho, c) + \sum_i \kappa_i G(x, y; x_i, y_i, s_i) - D(\rho; \lambda, \varphi) \tag{4}$$

The three terms represent:

- $F_q(\rho, c)$: baseline regime flow (internal dynamics)
- $\sum_i \kappa_i G$: feedback loops from market participants
- D: dissipation and stabilization

2.2 Regime Flow Decomposition

The baseline flow decomposes into three physical mechanisms:

$$F_q(\rho, c) = \mathcal{L}_q \rho + \mathcal{N}_q(\rho, c) - \eta_q \Delta \rho \tag{5}$$

Linear operator $\mathcal{L}_q$. Baseline regime flow—how the current regime would evolve absent external shocks. This is calibrated to historical regime durations.

Nonlinear term $\mathcal{N}_q$. Amplification effects—how market participants reinforce trends or reversals. This captures reflexivity.

Diffusion $-\eta_q \Delta \rho$. Spatial smoothing—how regime information propagates across the asset space. High η_q means fast cross-asset contagion.

2.3 Feedback Pressure Decomposition

The feedback term splits into trend-reinforcing and mean-reverting components:

$$\sum_i \kappa_i G(x, y; x_i, y_i, s_i) = \sum_{i:\kappa_i > 0} G - \sum_{i:\kappa_i < 0} G \tag{6}$$

Trend-reinforcing pressure ($\kappa_i > 0$): Momentum traders, trend-followers, FOMO. These amplify existing moves.

Mean-reverting pressure ($\kappa_i < 0$): Value traders, statistical arbitrage, contrarians. These push prices back toward equilibrium.

The net feedback $\kappa = \sum_i \kappa_i$ determines regime stability. When $\kappa > 0$, trends accelerate. When $\kappa < 0$, reversals become likely.

2.4 The Dissipation Operator

The dissipation term captures three stabilization mechanisms:

$$D(\rho; \lambda, \varphi) = \alpha \lambda \rho + \beta \varphi \rho + \gamma \rho^2 \tag{7}$$

Persistence drag ($\alpha \lambda \rho$): Friction that slows regime changes. High α means sticky regimes.

Shock dissipation ($\beta \varphi \rho$): How quickly the system absorbs external shocks. φ is the shock intensity.

Crowding/saturation ($\gamma \rho^2$): Nonlinear damping when too many participants are in the same trade. This prevents infinite trend extrapolation.

2.5 Level Discretization and Occupation Density

For computational implementation, discretize the regime space:

$$n(t) = \left\lfloor \frac{\mathcal{R}_t - \mathcal{R}_{\min}}{\Delta\mathcal{R}} \cdot (N_L - 1) \right\rfloor, \quad \mathcal{R}_t = \sum_{\tau<t} \tilde{r}_\tau/\hat{\sigma} \tag{8}$$

Creation and annihilation events track regime entries and exits:

$$\hat{a}_t^\dagger \equiv \mathbf{1}_{\{\Delta n_t>0\}} \cdot |\Delta n_t|, \quad \hat{a}_t \equiv \mathbf{1}_{\{\Delta n_t<0\}} \cdot |\Delta n_t| \tag{9}$$

The occupation density over window W:

$$P(n,t) = \frac{1}{W} \sum_{\tau=t-W+1}^{t} \delta_{n,n_\tau} \tag{10}$$

Trading signal. When occupation density concentrates in high-n states (extreme regimes), mean reversion becomes probable. When it's uniform, the trend continues.

3. Model III: $\mathtt{Lambda}$ Position Sizing

3.1 The Lambda Decomposition

Position size follows a multiplicative decomposition of four factors:

$$\mathcal{A} = \underbrace{e^{-\lambda_1\mathcal{T}-\lambda_2\mathcal{V}-\lambda_3\mathcal{R}}}_{\alpha:\text{position scalar}} \cdot \underbrace{(1+\gamma_0\mathcal{Y}\cdot\mathbf{1}_{Y>0.7})}_{\Gamma:\text{gamma hedge}} \cdot \underbrace{e^{-\phi_0\mathcal{R}^2}}_{\Phi:\text{factor decay}} \cdot \underbrace{\text{sgn}(\sigma_h - \kappa\mathcal{C})(1-\mathcal{C})}_{\Lambda:\text{directional tilt}} \tag{11}$$

3.2 Position Scalar (α)

The base position size decays exponentially with:

- $\mathcal{T}$: time in trade (age penalty)
- $\mathcal{V}$: current volatility (risk penalty)
- $\mathcal{R}$: realized drawdown (pain penalty)

Rule. When any of these exceeds its 90th percentile, position size drops by at least 50%. This is not conservative—it is correct risk management.

3.3 Gamma Hedge (Γ)

The gamma term provides convexity adjustment:

$$\Gamma = 1 + \gamma_0\mathcal{Y}\cdot\mathbf{1}_{Y>0.7} \tag{12}$$

When the gamma indicator $\mathcal{Y}$ exceeds 0.7, the position is in a high-convexity region. The hedge factor increases position size to capture the convex payoff, but only if the base scalar permits.

3.4 Factor Decay (Φ)

Factor exposure decays quadratically:

$$\Phi = e^{-\phi_0 \mathcal{R}^2} \tag{13}$$

When regime strength $\mathcal{R}$ is high, factor decay is severe—the signal is crowded and will mean-revert. When $\mathcal{R}$ is moderate, factors persist.

3.5 Directional Tilt (Lambda)

The directional component combines horizon volatility with concentration:

$$\Lambda = \text{sgn}(\sigma_h - \kappa\mathcal{C})(1 - \mathcal{C}) \tag{14}$$

Interpretation. When horizon volatility σ_h exceeds concentration-adjusted threshold $\kappa\mathcal{C}$, the tilt is positive (long). The $(1 - \mathcal{C})$ term reduces exposure as concentration increases—avoid crowded trades.

4. Model IV: The Unified Framework

4.1 The Geometric Structure

All three models embed in a single geometric structure. The price manifold $\mathcal{M}_t$ has metric:

$$g_{ij}(t) = \rho(x_i, x_j, t) \cdot \hat{\sigma}_{RV}(t, \Delta) \cdot \mathcal{A}(t) \tag{15}$$

The metric has three multiplicative components:

- ρ: regime density from Model II
- $\hat{\sigma}_{RV}$: realized volatility from Model I
- $\mathcal{A}$: position scalar from Model III

4.2 Unified Dynamics

The complete system evolves according to:

$$\frac{\partial g}{\partial t} = -2\text{Ric}(g) + \mathcal{L}_X g + \frac{1}{2}\dot{\rho} \otimes \hat{\sigma}^2 + g \cdot \frac{\dot{\mathcal{A}}}{\mathcal{A}} \tag{16}$$

The terms represent:

- $-2\text{Ric}(g)$: Ricci flow—the manifold evolves toward constant curvature
- $\mathcal{L}_X g$: Lie derivative along the price flow
- $\frac{1}{2}\dot{\rho} \otimes \hat{\sigma}^2$: regime-volatility coupling
- $g \cdot \frac{\dot{\mathcal{A}}}{\mathcal{A}}$: position scaling effect

4.3 Trading Implementation

Step 1: Compute realized volatility. Integrate equation (1) over the past 20 days. Normalize by the Euler characteristic of the correlation matrix.

Step 2: Update regime density. Evolve equation (4) one timestep. Compute feedback pressure from order flow data.

Step 3: Calculate position scalar. Evaluate equation (11) with current market state variables.

Step 4: Construct unified metric. Combine via equation (15).

Step 5: Compute geodesic positions. The optimal portfolio lies on the geodesic between current position and target, minimizing path length in the unified metric.

4.4 The Edge

Traditional quant models treat volatility, regime, and position sizing as separate problems. The unified framework recognizes they are the same problem viewed through different geometric projections.

When you compute volatility through curvature, regimes through reaction-diffusion, and positions through geodesics—all on the same manifold—the result is a system that adapts to structural breaks before they fully materialize.

This is the difference between reacting to a drawdown and anticipating it.

Markets are manifolds. Trade them geometrically.